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TOEPLITZ DETERMINANTS AND LOGARITHMIC COEFFICIENTS FOR A SUBCLASS OF STARLIKE FUNCTIONS

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Abstract. The paper mainly investigates the initial coefficients for the subclasses of starlike functions defined by using the Sine function involving α ($0 \leq \alpha < 1$). We obtain upper bounds for the second and third Toeplitz determinants whose elements are the initial coefficients. Also, we obtain upper bounds of initial logarithmic coefficients for the subclass of starlike functions of order α which are connected with the sine function. Parts of the results we obtained generalized and improved previous work.

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Conflict of Interests

The authors declare that they have no conflict of interest, regarding the publication of this paper.

Data Availability Statement

The authors declare that this research is purely theoretical and does not associate with any data.

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